ST BENEDICT’S COLLEGE



|  |  |  |  |
| --- | --- | --- | --- |
| **SUBJECT** | Advanced Programme Mathematics | **DATE** | 8 July 2014 |
| **GRADE** | 12 | **MARKS** | 300 |
| **EXAMINER** | Mrs M Povall | **MODERATOR** | Mrs H Rademeyer |
| **NAME** |  | **DURATION** | 3 Hours |
| **CLASS** |  |  |  |
|  |  |  |  |
| **QUESTION** **NUMBER** | **DESCRIPTION** | **MAXIMUM MARK** | **ACTUAL MARK** |
| **SECTION A** | **Algebra and Calculus** |  |  |
| **1** | Complex numbers and equations with complex roots | 13 |  |
| **2** | Partial fractions | 7 |  |
| **3** | Algebra | 25 |  |
| **4** | Split graphs, discontinuity and differentiability | 15 |  |
| **5** | Finding limits and derivatives | 44 |  |
| **6** | Exponential and ln graphs | 11 |  |
| **7** | Sketching functions | 34 |  |
| **8** | Methods of integration | 21 |  |
| **9** | Sectors and Newton’s method | 18 |  |
| **10** | Min and Max problems | 12 |  |
|  | Total Section A | 200 |  |
| **SECTION B** | **Statistics** |  |  |
| **1** | Probability Theory | 13 |  |
| **2** | Confidence Intervals | 20 |  |
| **3** | Probability Distributions | 14 |  |
| **4** | Probability density functions | 19 |  |
| **5** | Hypothesis Testing | 12 |  |
| **6** | Least Squares Regression line | 22 |  |
|  | Total Section B | 100 |  |

**READ THE FOLLOWING INSTRUCTIONS CAREFULLY:**

1. This question paper consists of 12 pages and a formula sheet is supplied.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers as the questions are numbered.
5. All the necessary working details must be clearly shown.
6. Approved non-programmable calculators may be used except where otherwise stated.
7. It is in your own interest to write legibly and to present your work neatly.

8. Where necessary, round off to **two** decimal places in Section A, and to **four** decimal places

 In Section B.

SECTION A

Question 1 13 marKs

1. Solve for , where ϵ : (5)
2. Determine a, b and c if and , given that

and . (8)

question 2 7 marks

Decompose the following expression into partial fractions:

question 3 25 marks

Solve for :

1. (8)
2. i) If and , find without the use of a calculator. (4)

 ii) Rewrite the following as a single logarithm: (2)



1. A catenary is the curve formed by a flexible cable with a uniform density if it is

 suspended between two points. Electricity cables and the cables of suspension bridges

 are hung in this shape. The equation of a catenary, suspended symmetrically around the

3

-3

**P ( d ; 2 )**

X

Y

 line is given as:

 where is the scale factor of the catenary.

 An electricity cable, suspended between two poles, 6 metres apart is represented by

 the catenary sketched below with and

1. Determine the minimum height the electricity cable hangs above the ground. (2)
2. Explain the importance of this minimum height in real life. (2)
3. P(d;2) is a point on the catenary. Determine the value of d , if the height of the electric cable at point P is 2m above the ground. Round off to TWO decimal places. (7)



question 4 15 marks

Consider the sketch of the following split-function:

X

Y

1. Determine and if they exist. (6)
2. Give the value(s) of for which is discontinuous, and state the type of

discontinuity. (3)

1. For which value(s) of will not be differentiable?

Provide explanations for each of your answers. (6)

question 5 44 marks

1. Without using a calculator, determine

 (5)

(5)

1. Determine for each of the following (it is not necessary to simplify answers):

i. ( 1 - (5)

 ii. (6)

 iii. (6)

1. i. Find if (7)

ii. Determine the equation of the tangent to this curve at the point (1;1) (4)

1. Given that determine a formula for (6)

Question 6 marks 11

1. Sketch the functions and its inverse function

 on the same system of axes, clearly indicating

 intercepts and asymptotes. (6)

**b)** What is the equation of (3)

**c)** What is the domain and range of(2)

Question 7 marks 34

Let

1. Calculate the x and y intercepts of ? (5)
2. Find the equations of all the asymptotes of (6)
3. Determine and simplify your expression. (5)
4. Calculate and determine the nature of all stationary points. (10)
5. Sketch the curve of clearly showing the above information. (8)

Question 8 marks 21

Determine the following and show all working out:

1. (6)
2. (8)
3. (7)

question 9 marks 18

 The diagram shows a sector of a circle with centre O and radius r, and a chord DC which subtends

 an angle of θ radians at O, where

r

O

D

C

B

A

A square ABCD is drawn.

1. If the area of the shaded segment is exactly one eighth of the area of the square show that:

 (8)

1. Evaluate if :

1. (2)
2. (2)
3. Hence, using a suitable initial approximation, use the Newton-Rhapson formula to find the value of θ correct to 4 decimal places. (6)

question 10 marks 12

 A man in a rowing boat at B is 2km from the nearest point on a beach, T, as illustrated in the diagram

 below. He wants to reach his house, H that is 6km from T. He will row to S and then walk to H. He

 rows at 3km/h and walks at 5km/h. TSH is a straight line.

S

B

x

2 Km

H

T

6 Km

**a)**  If BS= km show that the distance he must walk is given by SH km. (2)

**b)** Determine the value of so that he will reach his house in the shortest possible time.

 Hint: time = (10)



sECTION B

Question 1 13 marKs

The two events A and B are such that P(A )= 0,4 , P(B) = 0,24 and P(A l B) = 0,25.

1. Prove that the probability that both events occur is 0,06. (2)
2. Calculate the probability that:

 i) at least one of the events occur. (2)

 ii) exactly one the events occur. (3)

 iii) B occurs given A has occurred. (3)

 **c)** Are events A and B independent? Support your answer using calculations. (3)

Question 2 20 marKs

   

Litre bottles of wine are advertised to contain litre of wine. The contents of a random sample of 20

1 litre bottles were measured. It was found that the average fill was 0,982 litres with a standard deviation of 0,068 litres.

**a)** Find a 96% confidence interval for the mean amount of wine in 1 Litre bottles. (8)

**b)** Comment on the question: ‘ Do litre bottles of wine contain 1000ml on average?’ (2)

**c)** A crate contains 15 bottles of red wine, 10 bottles of white wine and 5 bottles of rose wine. If

 exactly 3 bottles are randomly taken from the crate (without replacement) what is the probability

 that each is a different colour/ type? (10)



question 3 14 marKs

1. At St Benedict’s College 40% of the learners play hockey. In a random sample of 12

learners find the probability that exactly 7 play hockey. (7)

**b)** Mrs Povall marked 90 Mathematics papers and 12 learners scored below 50%.

 If Mrs Sillman selects 5 papers at random to moderate, what is the probability that she

 will moderate at least one of the papers of the learners that scored below 50%. (7)

Question 4 19 marKs

 A random variable has a probability density function given by

**a)** Show that (8)

**b)** The mode of a continuous data set is the value of the random variable where it is the most

 dense, ie where the density function reaches its maximum value. Find the mode. (6)

**c)** Now find an equation to find the median of this density function. (5)

Question 5 12 marKs

The lives of a certain make of battery have a normal distribution with mean 30 hours and variance 2,54.

When a large consignment of these batteries is delivered to the store the quality control manager tests the lives of 8 randomly selected batteries. The mean life was 28.8hours. Test whether there is cause for complaint at a 3% significance level that the battery life was less than it should be.



question 6 22 marKs

A pupil loses his original bivariate data set for an experiment but finds the following recorded on a piece of paper:

* 

**a)** Show that there are 15 data points in the data set. (2)

**b)** Determine . (2)

**c)** Show that the least squares regression line of on in the form is

 (8)

**d)** Use this regression line to estimate the – value if = 54. (2)

**e)** Calculate the correlation coefficient by using

 Given that: and

 (4)

**f)** Describe the correlation between the variables. (2)

**g)** How accurate would the value estimated for in d) be if the value = 54 is interpolated.

 (2)

